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# PRINCIPLES AND PROSPECTS FOR MICRO HEAT PIPES

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## ABSTRACT/RESUME

The fabrication and purpose of very small wickless heat pipes are discussed. A theoretical analysis of their performance characteristics shows that they have nearly the heat transport capability expected by scaling down from conventional heat pipes. They appear attractive for applications calling for close temperature control, but having only modest cooling requirements.

KEYWORDS Heat Pipes

## I. INTRODUCTION

How small can a useful heat pipe be? The present extremes in miniaturization of engineering systems are to be found in very large scale integrated electronic circuits and, as a currently emerging development, in a variety of micromechanical devices. Some of these systems already present problems of effective heat dissipation and temperature control that will become ever more challenging as the inevitable trends of increased component density and more demanding standards of precision and reliability continue. Ordinary macroscopic heat pipes have, of course, already been used for external cooling of a variety of electronic devices. This paper tells how and why small heat pipes may serve as integral internal components in such systems to help solve these thermal problems.

My objectives are simply to discuss some basic principles of micro heat pipe structure and function, and to give an elementary theory of their general performance characteristics. No schemes for specific implementations will be offered, because the possibilities are so varied and speculative, especially in view of uncertainties in the future directions of the architectures of microelectronic and micromechanical devices.

I will first propose a definition of micro heat pipes and give a description in general terms of their application, structure and thermal environment. I will add a very brief account of some proven techniques for fabrication of the objects described. The static state of micro heat pipes is discussed in connection with the problem of controlling the amount of liquid in the pipe. Then, following a formulation of the basic micro heat pipe flow problem in mathematical terms, some assumptions are made to simplify the problem, and an approximate general solution is obtained. Finally, from the discussion of this solution, from some quantitative examples, and by comparisons with other possible techniques of thermal control, the reader may gain a general appreciation of the prospects for applications of micro heat pipes.

Commonly the diameter of a capillary pore in the wick of a heat pipe is much smaller than the characteristic minimum dimension of the vapor flow channel. The capillary pore is also smaller than a typical dimension of the overall liquid flow passage. (Some grooved heat pipes are nearly exceptions to this latter observation).

In principle, one may imagine a conventional heat pipe reduced in scale linearly in all dimensions to the point at which the capillary pores are but marginally bigger than the molecules of the working fluid, but practical difficulties of fabrication preclude the achievement of this ultimate in miniaturization. The smallest achievable heat pipe will have a total flow cross-sectional area near the lower limit of fabricability, with no conventional wick structure, but instead deriving its capillary flow-inducing properties from a noncircular cross section.

I define a <u>micro heat pipe</u> as one so small that the mean curvature of the vapor-liquid interface is necessarily comparable in magnitude to the reciprocal of the hydraulic radius of the total flow channel. The configurations of interest, under this definition, are channels of a few centimeters long with a convex but cusped cross section (as, for example, a polygon), having a diameter in the range of about 10 to 500 μm.

## II. APPLICATIONS, STRUCTURE AND FABRICATION OF MICRO HEAT PIPES

Currently, both microelectronic and micromechanical devices are all formed on single crystal silicon wafers. An excellent account of methods of housing, cooling, and interconnecting the silicon chips bearing large scale integrated circuits in digital computers has been given by Blodgett.[1] While these circuits require cooling, precise temperature control is not needed, because they perform satisfactorily within the nominal temperature range of 20 to 80°C. At present these devices are assembled in well-spaced planar arrays that permit interplanar access to conventional cooling methods, with the requirement therefore given on the basis of surface area. Cooling by forced flow of air can handle a maximum heat flux of about 2 W/cm$^2$ of working chip. For increased serial computational speed the trend is toward ever higher chip density. Blodgett[1] describes a modular planar packaging and cooling scheme using conduction through metal to an array of forced flow water channels that provides up to 20 W/cm$^2$ of cooling at the chip surface. The now emerging parallel processing computational technologies are likely to intensify the demand for ever larger cooling capacity, with the expectation that this must eventually be specified on a volumetric basis.

Angell, Terry, and Barth[2] give an account of how micromachining of silicon makes it possible to build mechanical devices almost as small as microelectronic ones. These include "..valves, springs, mirrors, nozzles, connectors, printer heads, circuit boards, heat sinks and sensors for properties such as force, pressure, acceleration and chemical concentration. Even a device as complex as a gas chromatograph, an instrument for identifying and measuring gases in an unknown mixture, can be built on a disk of silicon a few centimeters in diameter". While none of the devices require appreciable heat removal, the quantitative sensing devices may benefit from close temperature control in order to achieve accuracy in a variable environment. Various microsensors incorporating active microelectronic circuitry for local signal processing are under development, and these hybrid devices may need both heat removal and temperature control.

Tuckerman and Pease[3] have developed a micromechanical heat sink on the back of an integrated circuit silicon chip that can handle power dissipation of 1 KW/cm$^2$. It consists of parallel, subsurface 300- by 50-μm rectangular channels, with 100-μm interchannel spacing, cooled by the forced circulation of water. While this scheme achieves very high cooling capacity, it does not appear capable of providing good temperature uniformity over the area of the chip.

In order to achieve a high standard of uniformity and control while simultaneously providing substantial cooling capacity, I propose the use of micro heat pipes. The evaporator portion of the micro heat pipe configuration would be imbedded integrally in the substrate to be cooled, either as an array of separate parallel pipes or in a system of hierarchical branching patterns. The high thermal conductivity of silicon would provide good thermal coupling of the substrate to the pipes. The condenser section of the micro heat pipe configuration would be immediately external to the heat generation region of the substrate and would be cooled by close conductive coupling to a suitable gas or liquid forced convection system. The required length, cross sectional area, volume of working fluid and density of heat pipes will be determined jointly by the heat load to be dissipated and the desired uniformity of temperature. The control of the inventory of working fluid and its disposition throughout the pipe channels are matters of some importance, but discussion of these questions is best deferred until the theory of the heat transport capacity of micro heat pipes is developed below.

An impressive variety of techniques employed in the the fabrication of silicon micromechanical devices is described in Ref. 2. Three of these suffice to make micro heat pipe channels by the method now to be described.

The first procedure is the preparation of an oxide mask on the surface of the silicon wafer using photolithography. Here the entire surface of the chip is covered with an oxide layer by heating in an atmosphere of steam; the oxide layer is coated with a thin layer of organic photoresist, which is then exposed to ultraviolet radiation through a photomask pattern of the heat pipe channel layout; the surface is treated with solvent to remove the unexposed photoresist; finally, the oxide layer that is not covered by protecting photoresist is removed by solution in hydrofluoric acid, and then the photoresist is stripped off with sulfuric acid that does not attack the oxide or the silicon. The second procedure is the formation of channels in the surface using an anisotropic etchant that attacks the silicon at openings in the oxide layer. Anisotropic etchants are special acid mixtures that etch at different rates in different directions in the crystal lattice. They can form well-defined cavity shapes with sharp edges and corners, without undercutting the oxide mask. In particular, channels of sharply defined triangular cross section can be obtained. The final procedure consists of enclosing the channels by stripping the oxide off the surface and sealing the surface of the wafer to a thin plate of Pyrex glass by anodic bonding, a special hot electrostatic process.

The micro heat pipes will, of course, require fill tubes for loading with a measured amount of working fluid. One method of fabrication of a micro valve suitable for final closure of the heat pipes is described in detail in Ref. 2.

## III. STATIC BEHAVIOR AND CONTROL OF LIQUID VOLUME

As will be shown, the heat transport capacity of micro heat pipes depends more critically on the volume of working fluid available than do conventional heat pipes with wicks. It is therefore desirable either to load the pipe accurately with nearly optimum amount of working fluid, or to provide a measure of passive control of the quantity of fluid in the pipe itself by providing a suitable sump.

Before discussing this question, we recall some properties of static capillary behavior of liquid in a closed cavity. I will assume that the liquid wets the wall of the cavity and that gravity is negligible; that is, the maximum pressure difference between two points in the liquid is small compared to the difference in pressure between the liquid and vapor phases due to surface tension. At each point on the vapor-liquid interface, the surface has two principal radii of curvature, $R_1$ and $R_2$, in planes at right angle to one another and normal to the surface. The mean radius of curvature, R, is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad . \tag{1}$$

If the surface tension is $\sigma$ and the pressures in liquid and vapor are $P_L$ and $P_V$ respectively, then

$$P_V - P_L = \frac{\sigma}{R} \quad . \tag{2}$$

When the system is at rest, the mean radius of curvature of the interface therefore will be the same everywhere. As the cavity is filled with liquid, R will start small, with only corners and grooves containing liquid, and will then pass through a maximum value, with the vapor thereafter located in a liquid-enclosed bubble with diminishing R as the liquid more nearly fills the entire cavity. The position of such a bubble in the cavity, if not uniquely determined by capillary force, will be fixed by gravity, however small. The maximum value of the che mean radius of curvature is a useful capillary characteristic dimension, depending only on the size and shape of the cavity. As we shall see, the optimum volume fraction of liquid is always less than that associated with the maximum mean capillary radius of curvature.

A micro heat pipe can be accurately supplied with liquid with the help of a microflask of just the desired volume. This is to be connected to the pipe cavity through a low volume on-off microvalve, and similarly to a liquid reservoir and to a vacuum line. The microflask should have a maximum mean capillary radius of curvature that is large compared to that of the micro heat pipe. The pipe and flask are first evacuated and then closed from the vacuum line. The flask is then filled completely from the reservoir and again is closed. Upon opening the empty pipe to the full flask, the fluid automatically flows to the micro heat pipe by capillary action, after which the connecting valve is closed.

The micro heat pipe may be provided with a sump to allow some variation in the amount of fluid in the pipe as the heat load varies. The sump is best placed at the terminal end of the condenser section, and with respect to gravity, below the entire heat pipe. The shape and size of the sump cavity must be chosen with due consideration of how the fluid will apportion itself between the pipe and the sump. The sump by itself should have a maximum mean capillary radius that is smaller than that of the pipe, otherwise the pipe may flood by capillary transfer of fluid to it from the sump. If, however, the maximum mean capillary radius of the sump is too small, this may unduly deplete the heat pipe of working fluid.

## IV. THE STEADY STATE FLOW PROBLEM

In order to concentrate on the one essential new feature that must be understood in this first discussion of micro heat pipes, I will make certain simplifying assumptions. The inclusion in the theory of a number of the effects thus eliminated will eventually be necessary when actual devices are to be designed, but to assess the general prospects for micro heat pipes these presumably minor corrections are not required. We suppose that the heat pipe is very long compared to its maximum diameter. If the pipe is not straight, let the radius of curvature of the pipe axis be large compared to its diameter, or if small anywhere, let such bends be concentrated in a very small fraction of the total length of the pipe. Let the vertical extent of the pipe be small compared to the height of capillary rise of the working liquid. I assume that the shape and area of the pipe cross section are specified, and if these are not constant along the pipe, let the fractional change over an axial distance equal to a pipe diameter be small. Assume that the local axial heat transport is also specified, that this varies but slowly along the pipe, and is constant in time. Finally, assume that the vapor and liquid are each essentially at constant temperature and density.

With these assumptions the fluid moves as in steady-state incompressible gravity-free parallel flow. The equation of motion of the fluid is

$$\mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = \frac{dP}{dz} \quad . \tag{3}$$

The viscosity here is that for liquid or vapor as appropriate. The pressures in liquid and vapor are different, but each varies only with z. The desired solution of Eq. (3) must satisfy certain boundary conditions. At the cavity wall we must have w=0. Both the flow velocity and the tangential sheer stress must be continuous across the interface. The condition to be satisfied on the component of the force normal to the interface is just Eq. (2). The interface between the vapor and liquid will locally have the form of a sector of a circular cylinder of radius, R, which varies slowly with z. Since R(z) is not known a priori, we are faced with a novel free-surface problem. For the present discussion, detailed but necessarily numerical solutions of the foregoing problem would be less instructive than an approximate general solution.

## V. AN APPROXIMATE GENERAL SOLUTION

The vapor and liquid phases will be in countercurrent flow and the common velocity at their interface will ordinarily be small compared to the mean velocity of either phase. I will assume that the interface velocity is zero. This introduces some error but greatly simplifies the problem, for it permits the use of the scaling law now to be derived.

Consider the flow of fluid (either the liquid or the vapor) satisfying Eq. (3) in the interior of a region of local cross-sectional area A, and having w=0 on its boundary. Transform to new dimensionless variables x', y' and w' given by:

$$x = A^{1/2}x' ; \quad y = A^{1/2}y' ; \quad w = -\frac{A}{\mu}\frac{dP}{dz}w' \quad . \tag{4}$$

Then w' satisfies

$$\frac{\partial^2 w'}{\partial x'^2} + \frac{\partial^2 w'}{\partial y'^2} = -1 \quad , \tag{5}$$

in a similarly shaped region of unit area, with w'=0 along its boundary. Now the local mass flow is

$$\dot{m} = \rho \iint w \, dxdy \quad , \tag{6}$$

where the integral is taken over the area A. Using the transformation (4) in the integrand of (6) gives

$$\dot{m} = -\frac{K\rho A^2}{8\pi\mu}\frac{dP}{dz} \quad . \tag{7}$$

Here the quantity K is defined by an integral over a shape of unit area,

$$K = 8\pi \iint w' dx' dy' \quad . \tag{8}$$

K is dimensionless, takes the value one for a circular disc, and is less than one for any other shape.[4] A short table of values of K from Ref. 4 is given in Table I.

The two equations obtained by rewriting Eq. (7) for the liquid and vapor cases respectively, together with Eq. (2), give three equations for the determination of $P_V(z)$, $P_L(z)$ and R(z). If Eq. (2) is differentiated with respect to z, and Eq. (7) then used to eliminate $P_V$ and $P_L$, one obtains

TABLE I

SOME VALUES OF THE FLOW SHAPE FACTOR

| Shape | K |
|---|---|
| Circle | 1.000 |
| Regular hexagon | 0.964 |
| Square | 0.883 |
| Triangle, 60°-60°-60° | 0.725 |
| Triangle, 45°-45°-90° | 0.656 |
| Triangle, 30°-60°-90° | 0.597 |

$$\frac{8\pi\nu_V\dot{m}_V}{K_V A_V^2} = \frac{8\pi\nu_L\dot{m}_L}{K_L A_L^2} - \frac{d}{dz}\frac{\sigma}{R} \quad . \tag{9}$$

In the steady state the local liquid and vapor mass flows are equal in magnitude and proportional to the local heat flux, so we can write

$$\dot{m}_V(z) = -\dot{m}_L(z) = \frac{\dot{Q}}{\lambda}h(z/L) \quad . \tag{10}$$

The total pipe cross section, A(z), being given we have

$$A(z) = A_L(z) + A_V(z) \quad . \tag{11}$$

It is useful to relate $A_L$ and R with a geometrically determined dimensionless factor, β, by

$$A_L = \beta^2 R^2 \quad . \tag{12}$$

Using Eqs. (10), (11) and (12) in (9) yields the following first order ordinary differential equation for R(z):

$$\frac{dR}{dz} = \frac{8\pi\dot{Q}R^2 h(z/L)}{\sigma\lambda}\left[\frac{\nu_V}{K_V\left(A - \beta^2R^2\right)^2} + \frac{\nu_L}{K_L\beta^4R^4}\right] \quad . \tag{13}$$

For a general A(z), this equation would have to be integrated numerically, but if A is constant, the equation is separable and therefore immediately reducible to quadratures. The factors $K_V$ and $K_L$ may depend on R. This dependence can be found by solving Eq. (5) with the shapes implied by the specific geometry, then calculating Eq. (8). However, as suggested in the table above and as discussed in Ref. 5, substantial variations in shape of cross section make only small changes in the magnitude of K. The factor $\beta$ may also depend on R. In a most important case, however, when the interface meniscus is a concave, circular, cylindrical sector intersecting at a fixed angle with two plane walls meeting in a corner, then $\beta$ is a constant.

For A, $K_V$, $K_L$, and $\beta$ all constant, the solution of Eq. (13) depends only on two groups of parameters. We define dimensionless distance along the pipe, $\zeta$, and interface radius of curvature, $\phi$, by

$$\zeta = z/L \quad , \quad \phi = \beta A^{-1/2} R \quad . \tag{14}$$

and dimensionless kinematic viscosity and heat flux parameters by

$$\alpha = \frac{K_L \nu_V}{K_V \nu_L} \quad \text{and} \quad \gamma = \frac{8\pi \dot{Q} \nu_L L}{\beta\sigma\lambda K_L A^{3/2}} \quad . \tag{15}$$

Using Eqs. (14) and (15), Eq. (13) reduces to the dimensionless form,

$$\frac{d\phi}{d\zeta} = \gamma h(\zeta)\left[\frac{\alpha\phi^2}{\left(1 - \phi^2\right)^2} + \frac{1}{\phi^2}\right] \quad . \tag{16}$$

Upon separating the variables and integrating, the solution of Eq. (16) can be written

$$\gamma H(\zeta) = G(\phi;\alpha) - G(\phi_0;\alpha) \quad , \tag{17}$$

where

$$H(\zeta) = \int_0^{\zeta} h(\zeta)d\zeta \quad , \tag{18}$$

and

$$G(\phi;\alpha) = \int_0^{\phi} \frac{\phi^2(1 - \phi^2)^2 d\phi}{\alpha\phi^4 + \left(1 - \phi^2\right)^2} \quad . \tag{19}$$

The constant of integration, $\phi_0$, is determined implicitly from the known total amount of liquid present in the heat pipe. The fraction, F, of heat pipe volume occupied by liquid is

$$F = \frac{1}{L}\int_0^L \frac{A_L}{A}\, dz = \int_0^1 \phi^2(\zeta)d\zeta \quad . \tag{20}$$

This can be satisfied for given F only by the correct choice of $\phi_0$ in Eq. (17).

A family of curves of the function $G(\phi;\alpha)$ is plotted in Fig. 1. The variable $\phi$ may take only values in the interval, $0<\phi<1$, which are physically allowed by the shape of the heat pipe cavity, and are consistent with unflooded heat pipe behavior. If the cavity has one or more sharp corners, $\phi$ may approach zero, but $\phi$ may not exceed a value, $\phi_{max}$, corresponding to the maximum mean capillary radius for the cavity, as explained in Sect. III.

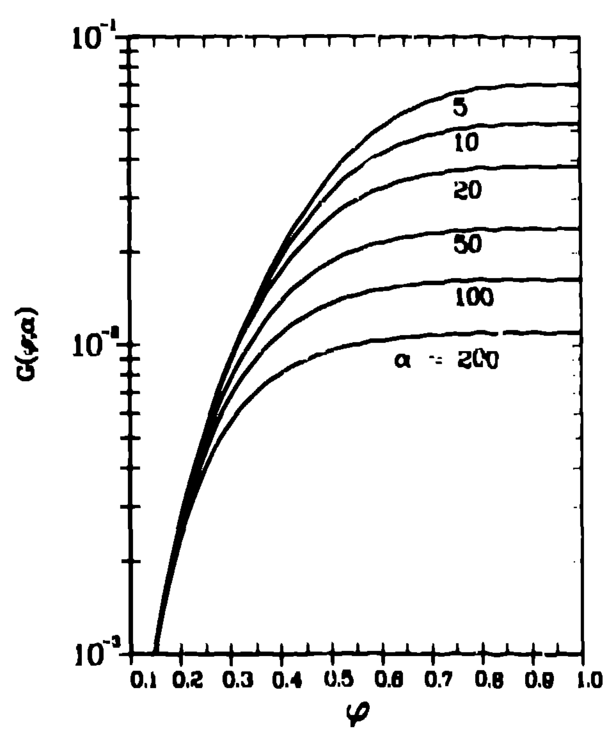


**FIGURE 1 Values of $G(\varphi;\alpha)$**

The maximum heat transport of a pipe is attained for the largest constant value of $\gamma$ for which Eq. (17) has a physically allowed solution for all $0<\zeta<1$. If $\phi_{max}$ is taken as corresponding to the minimum mean radius of curvature of the cavity surface, this maximum, $\gamma_{max}$, is

$$\gamma_{max} = [G(\phi_{max};\alpha) - G(\phi_{min};\alpha)]/H(1) \quad . \qquad (21)$$

For properly designed micro heat pipes, this expression is nearly independent of the details of the geometry. We will have $G(\phi_{max};\alpha)$ only slightly smaller than $G(1;\alpha)$, and $G(\phi_{min};\alpha)$ only slightly larger than zero. For the common case of a uniformly heated evaporator, no adiabatic section, and a uniformly cooled condenser, $H(1)=1/2$. Then $\gamma_{max} \simeq 2G(1;\alpha)$. In further simplification, for $\alpha$ in the range $5<\alpha<200$, one finds that with error not exceeding 10%, $G(1;\alpha) \simeq 0.16\alpha^{-1/2}$.

## VI. PERFORMANCE CHARACTERISTICS

Using the approximations of the last paragraph of Sect. V, and returning to the physical quantities of Eq. (15), one obtains

$$\dot{Q}_{max} = \left(\frac{0.16\beta(K_L K_V)^{1/2}}{\beta_* H(1)}\right)\left(\frac{\sigma\lambda}{\nu_L}\right)\left(\frac{\nu_L}{\nu_V}\right)^{1/2}\left(\frac{A^{3/2}}{L}\right) . \qquad (22)$$

The maximum heat transport for a conventional heat pipe with a wick, optimized in respect to wick pore size and wick cross-sectional area, was derived in Ref. 6. Except for the numerical and geometric shape quantities in the first factor, the result is identical with Eq. (22). The second factor in Eq. (22) is the well-known characteristic heat flux, which measures the quality of the liquid as a heat pipe fluid. The third factor depends on the kinematic viscosity ratio, and varies mainly with the vapor density. The final factor indicates that the maximum heat flux is proportional to the cross-sectional area of the pipe, and to the ratio of its diameter to its length. Using the results from Ref. 6, the ratio of the maximum heat flux of a wickless micro heat pipe to that of an optimized heat pipe with wick, $\dot{Q}_w$, is

$$\frac{\dot{Q}_{max}}{\dot{Q}_w} = 0.08\beta\left(\frac{hK_L K_V}{\epsilon}\right)^{1/2} \quad . \qquad (23)$$

All of this means that, aside from a simple numerical factor and the extreme difference in typical size, the design considerations for wickless micro heat pipes are essentially the same as those for conventional wicking limited heat pipes.

As an illustrative case, consider a micro heat pipe of equilateral triangular cross section filled with the optimum amount of methanol ($CH_3OH$), to operate at 50°C with a uniformly heated evaporator and uniformly cooled condenser. The derived properties for this case are given in Table II. The cross section of the vapor space varies from an equilateral triangle (K=0.725) at the beginning of the evaporator to an inscribed circle (K=1.0) at the end of the condenser; the assumed tabulated value is the simple average of these. The liquid cross section is of constant shape ($\beta$=1.43), but the tabulated value of $K_L$ is only a rough estimate for this shape. The maximum heat transport is obtained by filling 18% of the volume of the pipe with liquid. Calculated from Eq. (17), the fraction of the cross section occupied by liquid as a function of position along the pipe is plotted in Fig. 2.

If the triangular pipe has a side of 0.02 cm and a length of 1 cm, then using values from Table II in Eq. (22) one finds $\dot{Q}_{max}$ = 0.03 W. For comparison, this is 40% of the value for an optimized heat pipe with wick, as calculated from Eq. (23) using representative wick parameters, h=15 and $\epsilon$=0.5.

If placed closely in parallel on a surface, micro heat pipes of this size could provide a few watts per square centimeter of cooling. If imbedded in a solid as a matrix of parallel pipes to

TABLE II

DERIVED PROPERTIES OF AN EQUILATERAL TRIANGULAR METHANOL MICRO HEAT PIPE AT 50°C

| | |
|---|---|
| $P_V$ = 400 mm Hg | $\phi_{max}$ = 0.63 |
| $\sigma\lambda/\nu_L$ = 4.4 MW/cm$^2$ | $K_V$ = 0.86 |
| $\nu_L/\nu_V$ = 0.07 | $K_L$ = 0.5 |
| $\alpha$ = 24 | $H(1)$ = 0.50 |

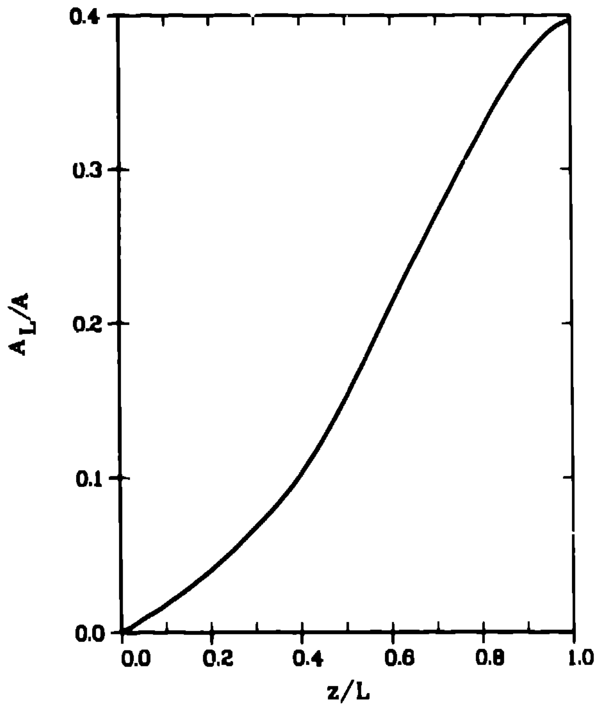


**Figure 2 – Liquid cross section fraction versus distance along pipe.**

the extent of, say 10% by volume, they could provide a few tens of watts per cubic centimeter of cooling. As cited in Sect. II, such cooling rates are readily exceeded by forced convection systems.

Like heat pipes of ordinary size, micro heat pipes will exhibit an extremely small temperature gradient even when working at nearly maximum heat transport capacity. The capability to produce a region of uniform temperature and to hold it constant in time in an environment of nonuniform and varying thermal load is uniquely characteristic of two-phase heat transport systems. When there is, in addition, a particular requirement for a self-acting closed system, then the microheat pipe may be the unique method of choice.

## VII. CONCLUSION

The foregoing performance analysis of wickless micro heat pipes shows that they are candidates for the thermal control of micromechanical and microelectronic devices in cases where thermal loads are modest, and where a high standard of temperature constancy and uniformity are required.

## NOTATION

(Defining equation number given in parenthesis)

| | |
|---|---|
| $A$ | cross-sectional area |
| $b$ | wick pore geometric factor |
| $e$ | liquid volume fraction of wick |
| $F$ | liquid volume fraction of pipe (20) |
| $G$ | interface curvature integral (19) |
| $h$ | fraction of total heat transport (10) |
| $H$ | integral of h (18) |
| $K$ | flow shape factor (8) |
| $L$ | length of heat pipe |
| $\dot{m}$ | mass flow rate |
| $P$ | pressure |
| $\dot{Q}$ | heat flow rate |
| $R$ | radius of curvature |
| $w$ | z-component of velocity |
| $x,y$ | coordinates orthogonal to z and to each other |
| $z$ | coordinate measured along pipe axis |
| $\alpha$ | kinematic viscosity parameter (15) |
| $\gamma$ | heat transport parameter (15) |
| $\beta$ | liquid geometric shape parameter (12) |
| $\zeta$ | dimensionless axial distance (14) |
| $\lambda$ | heat of vaporization |
| $\mu$ | viscosity |
| $\nu$ | kinematic viscosity = $\mu/\rho$ |
| $\rho$ | density |
| $\sigma$ | surface tension |
| $\phi$ | dimensionless radius of curvature (14) |

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